Robustness Criteria as a Framework to Capture Students' Algebraic Understanding Through Contextual Algebraic Tasks

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Overview and Rationale

As part of the Algebra Teaching Study (www.ATS.berkeley.edu) we've modified tasks and developed rubrics to capture students' robust understanding of algebra as evidenced by their work on contextual tasks.

We hypothesize that in order to solve contextual algebraic tasks, students need to demonstrate the understandings covered by our Robustness Criteria (RCs). We are trying to capture a variety of algebra thinking and skills at a finer level of detail than typically provided by scoring rubrics, which focus on the right answer and/or procedure (e.g., those available through the Mathematics Assessment Resource Service, www.noycefdn.org/resources.php).

This poster presents a subset of our work in which we address the following question: **Can we develop scoring rubrics that capture students' algebraic understanding at the level of our five criteria for robust understanding?**

Robustness Criteria (RCs)

With Links to Relevant Literature

Five criteria are used to assess growth in students' abilities to solve contextual problems in our assessment:

1) Interpreting Context: Students are able to interpret a problem statement to make sense of the problem situation.
- **Walkington, Sherman, & Petrosino, 2012**
- **Common Core State Standards for Mathematics ([CCSS-M], CCSS-I, 2010), Standard 1 for Mathematical Practice**

2) Identifying Relevant Quantities and Articulating Relationships: Students are able to (2a) identify which quantities are relevant to the problem situations and (2b) articulate the mathematical relationships between quantities.
- **National Council of Teachers of Mathematics (NCTM, 2000) Algebra 1 Content Standards**
- **Driscoll (1999) *Fostering Algebraic Thinking***
- **CCSS-M (2010) Standard 2**

3) Representing Quantitative Relationships: Students are able to (3a) generate appropriate algebraic representations, and (3b) interpret and make connections between representations.
- **CCSS-M (2010) Standard 4**
- **NCTM (2000) Understanding Patterns, Relations, and Functions Standard**

4) Executing Procedures and Checking Solutions: Students are able to (4a) execute algebraic procedures and arithmetic calculations and (4b) check the plausibility of their results by attending to the problem context and considering their solution methods.
- **NCTM (2000) Problem Solving Standard**
- **CCSS-M (2010) Practices 6 and 7**

5) Explaining and Justifying Reasoning: Students are able to clearly and thoroughly explain and justify their reasoning.
- **NCTM (2000) Reasoning and Proof Standard**
- **CCSS-M (2010) Practice 3**

Method and Design

Student Data
- Students completed pre- and post-assessments consisting of tasks that provide opportunities for students to explain and justify reasoning. In this poster, we are only using the post-test data.
- Rubrics were created and used to score student work that captured evidence of students' robust understanding.

Validity and Reliability
- **Validity:** The correctness scores of our assessments are highly correlated with MARS rubrics with overall correlation higher than 0.7, and moderately highly correlated with holistic scoring ($r > 0.6$) at the RC level.
- **Inter-rater reliability**: Our inter-rater reliability is substantially high. All correlations between double coders are above $r = 0.75$ except for RC 5, which was 0.6 due to the scoring differences on a single assessment.

Sample Task and Student Work

Going Bowling

(Adapted from Mathematics Assessment Resource Service, http://www.noycefdn.org/resources.php, copyright 2003)

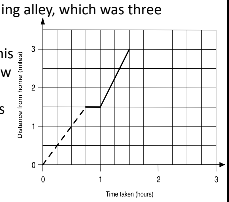
Craig and James walked from their house to the bowling alley during their summer vacation.

The time/distance graph below shows their trip to the bowling alley, which was three miles away. They left home at 10:30am.

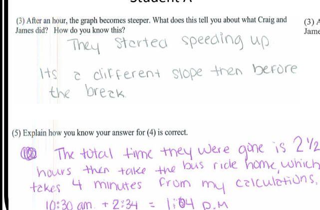
(3) After an hour, the graph becomes steeper. What does this tell you about what Craig and James did? How do you know this?

(4) Craig and James bowled for an hour and then took a bus home. The bus averaged 12 miles per hour. What time did they get home? Show your work.

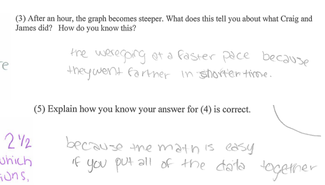
(5) Explain how you know your answer for (4) is correct?



Student A



Student B



Scoring Rubrics and Analysis

Task Sub-parts	Solution	Associated RC	Strategy (as appropriate)	Score	A	B
Part 1	Correct answer (stopped and rested, took a break, etc.)	3b, 4b		1	1	1
	Correct explanation	5		1	1	0
		4b	Connects to context of the story	1		
		2a,2b, 3b	Articulates relationship between distance and time via the horizontal line in graph	1		
Part 2	2 mph. **Partial credit (1)**: e.g. 1.5 miles in 45 minutes.	2a, 3b		2	1	1
		3a	Extends line to 1 hr and reads distance for that time	1		
		4a	Uses points from graph to calculate slope or r=d/t	1		
Part 3	Walked faster.	2a,3b,4b		1	1	1
	Explanation	5		1	1	1
		2b	Recognizes meaning of slope in this context - relationship between distance traveled in a certain amount of time	1	1	1
Part 4	(1:15 p). **Partial credit (1)**: Shows some correct work.	2a,3b,4a,4b		2	1	2
Part 5	Explains solution	5		1	1	0
	(more than 1 strategy may apply)	4a	References calculations for time	1	1	
		3a,3b	Draws a table and references it in explanation	1		
		3b	Reasons from the graph (I.e. completed part 6 first)	1		
		4b	Draws on context (relates answer to problem scenario)	1	1	
Part 6	Correct addition to graph. **Partial credit (1)**	3a		2	1	2
	RC 1: Are students answers within the scope of the problem?	1		2	2	2

The two students got the same overall *correctness score*, which indicates the same level of mastery using a standard rubric that captures the correctness of each part.

Correctness	11	8	8
RC1	2	2	2
RC2a	4	2	2.5
RC2b	2	1	1
RC2b associated with RC5		1	1
RC3a	3	0.5	1
RC3b	7	3	3.5
RC4a	3	1.5	1
RC4a associated with RC5		1	0
RC4b	5	3.5	3
RC4b associated with RC5		1	0
RC5	3	3	1

Our rubrics offer a more nuanced view of student understanding at the RC-level. We can see the two students have different types and levels of algebraic understanding. Student A scored higher on explaining and justifying what s/he did mathematically and connect it back with the context, while student B did fairly well on making sense of the task and representing the quantities without explain and justify what s/he did.

Conclusions and Future Direction

We have a scoring tool that is efficient, valid, and fairly reliable for assessing the understanding of individual students. Our rubrics provide a reliable, fine-grained perspective on student understanding at the classroom level, making it possible to link student learning with instructional practices. This general approach to scoring student work on open-ended tasks could be revised to score assessments on other mathematics topics (e.g., geometry).

References

Common Core State Standards Initiative (2010). *Common Core State Standards*. Washington D. C.

Driscoll, M. (1999). *Fostering algebraic thinking: A Guide for teachers grades 6-10.* Portsmouth, NH: Heinemann.

NCTM (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

Walkington, C., Sherman, M., & Petrosino, A. (2012). 'Playing the Game' of Story Problems: Coordinating Situation-based Reasoning with Algebraic Representation. *Journal of Mathematical Behavior, 31*, 174–195.