What Makes for Powerful Classrooms – and What Can We Do, Now That we Know?

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1983:

PROBLEM SOLVING
IN THE
MATHEMATICS CURRICULUM

A Report, Recommendations, and
An Annotated Bibliography

Alan H. Schoenfeld

MAA Notes Number 1

THE MATHEMATICAL ASSOCIATION OF AMERICA
Committee on the Undergraduate Program in Mathematics
Today’s Agenda

1. What really matters in math classrooms? (An intro to the TRU framework)
2. Tools for supporting powerful classroom instruction:
   - Formative Assessment lessons
   - Planning and Reflection
3. What can we do? Some thoughts...
Part 1:
What matters in classrooms?
If you had 5 things to focus on in order to improve mathematics teaching, what would they be?
Why 5 (or fewer)?

If you have too many things to work on, it’s difficult to keep all of them in your mind.
What properties should those 5 things have?

They’re necessary and sufficient (there’s nothing essential missing).

They each have a certain “integrity” and can be worked on in meaningful ways.
Welcome to the Teaching for Robust Understanding of Mathematics (TRU Math) framework
If we had the whole morning, we would look at a bunch of videos and discuss what we see in them.
But we don’t. So, I’ll show you one 6th grade teaser and ask you to think about the wide range of classrooms you’ve seen, pre-school to graduate school.
The clip: A 6th grade classroom in an inner city, low income Chicago school.

The context: a “Formative Assessment Lesson” entitled “translating between fractions, decimals, and percents.”
This is one of 100 “Formative Assessment Lessons” available for free at Mathematics Assessment Project website. To date we have more than 5,000,000 downloads.

I’ll say more about these later.
<table>
<thead>
<tr>
<th>Decimals</th>
<th>Percents</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>____.%</td>
</tr>
<tr>
<td>0.05</td>
<td>____%</td>
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<tr>
<td><em>.</em>_</td>
<td>80%</td>
</tr>
<tr>
<td>0.375</td>
<td><em>.</em>_</td>
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<td>____%</td>
<td>12.5%</td>
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<td>0.75</td>
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<td>1.25</td>
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<td>____%</td>
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</tr>
<tr>
<td><em>.</em>_</td>
<td>____%</td>
</tr>
</tbody>
</table>
Take turns to:

1. Fill in the missing decimals and percents.

2. Place a number card where you think it goes on the table, from smallest on the left to largest on the right.

3. Explain your thinking.

4. The other members of your group must check and challenge your explanation if they disagree.

5. Continue until you have placed all the cards in order.

6. Check that you all agree about the order. Move any cards you need to, until everyone in the group is happy with the order.
## Areas

<table>
<thead>
<tr>
<th>Area A</th>
<th>Area B</th>
<th>Area C</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Area A" /></td>
<td><img src="image2.png" alt="Area B" /></td>
<td><img src="image3.png" alt="Area C" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Area D</th>
<th>Area E</th>
<th>Area F</th>
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</thead>
<tbody>
<tr>
<td><img src="image4.png" alt="Area D" /></td>
<td><img src="image5.png" alt="Area E" /></td>
<td><img src="image6.png" alt="Area F" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Area G</th>
<th>Area H</th>
<th>Area I</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image7.png" alt="Area G" /></td>
<td><img src="image8.png" alt="Area H" /></td>
<td><img src="image9.png" alt="Area I" /></td>
</tr>
</tbody>
</table>
# Fractions

<table>
<thead>
<tr>
<th>Fraction 1</th>
<th>Fraction 2</th>
<th>Fraction 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{8}$</td>
<td>$\frac{4}{5}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td>$\frac{6}{10}$</td>
<td>$\frac{5}{4}$</td>
</tr>
<tr>
<td>$\frac{1}{8}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Scales

Scale A

Scale B

Scale C

Scale D

Scale E

Scale F

Scale G

Scale H

Scale I
Take turns to:

1. Match each area card to a decimals/percents card.

2. Create a new card or fill in spaces on cards until all the cards have a match.

3. Explain your thinking to your group. The other members of your group must check and challenge your explanation if they disagree.

4. Place your cards in order, from smallest on the left to largest on the right. Check that you all agree about the order. Move any cards you need to, until you are all happy with the order.
## The complete answer set (decimals, %, fractions, area, measure)

<table>
<thead>
<tr>
<th>Decimal</th>
<th>%</th>
<th>Fraction</th>
<th>Area</th>
<th>Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>5%</td>
<td>( \frac{1}{20} )</td>
<td><img src="image1" alt="Area Image" /></td>
<td><img src="image2" alt="Measure Image" /></td>
</tr>
<tr>
<td>0.125</td>
<td>12.5%</td>
<td>( \frac{1}{8} )</td>
<td><img src="image3" alt="Area Image" /></td>
<td><img src="image4" alt="Measure Image" /></td>
</tr>
<tr>
<td>0.2</td>
<td>20%</td>
<td>( \frac{1}{5} )</td>
<td><img src="image5" alt="Area Image" /></td>
<td><img src="image6" alt="Measure Image" /></td>
</tr>
<tr>
<td>0.375</td>
<td>37.5%</td>
<td>( \frac{3}{8} )</td>
<td><img src="image7" alt="Area Image" /></td>
<td><img src="image8" alt="Measure Image" /></td>
</tr>
<tr>
<td>0.5</td>
<td>50%</td>
<td>( \frac{1}{2} )</td>
<td><img src="image9" alt="Area Image" /></td>
<td><img src="image10" alt="Measure Image" /></td>
</tr>
<tr>
<td>0.6</td>
<td>60%</td>
<td>( \frac{6}{10} )</td>
<td><img src="image11" alt="Area Image" /></td>
<td><img src="image12" alt="Measure Image" /></td>
</tr>
<tr>
<td>0.75</td>
<td>75%</td>
<td>( \frac{3}{4} )</td>
<td><img src="image13" alt="Area Image" /></td>
<td><img src="image14" alt="Measure Image" /></td>
</tr>
<tr>
<td>0.8</td>
<td>80%</td>
<td>( \frac{4}{5} )</td>
<td><img src="image15" alt="Area Image" /></td>
<td><img src="image16" alt="Measure Image" /></td>
</tr>
<tr>
<td>1.25</td>
<td>125%</td>
<td>( \frac{5}{4} )</td>
<td><img src="image17" alt="Area Image" /></td>
<td><img src="image18" alt="Measure Image" /></td>
</tr>
</tbody>
</table>
The classroom video went here. In it you see 6th grade students arguing seriously about the meanings of different representations for fractions and how they compare in magnitude.
Every time a group looks at videos, there are lots of comments about what the teacher is doing, and what it must feel like to be a student in that class.
And every time, it is easy to organize everything they say into five categories:
The Mathematics

Is it important, coherent, connected? Where are the big ideas? Are there opportunities for thinking and problem solving?
Cognitive Demand

Do the students have opportunities for sense making – for “productive struggle,” engaging productively with the mathematics?

- Surface questions
  - Tasks allowed for st. discussion
    - Structure t, s-s, t-s
  - Representations (multiple)
    - Support st discussion
    - Nature of activity is important
  - Dialogue supports exploration of misconceptions
  - Is lesson making meaning
    - For kids > size of math
    - “Chunk” = 1 strategy
    - Connections = reasoning

Cognitive Demand
Access and Equity

Who participates, in what ways? Are there opportunities for every student to engage in sense making?
Agency and Identity

Do students have the opportunities to do and talk mathematics? Do they come to see themselves as “math people,” or people who cannot do mathematics?
Formative Assessment

Does classroom discussion reveal what students understand, so that instruction can be adjusted for purposes of helping students learn?
These are the five dimensions of Teaching for Robust Understanding of Mathematics, or ...

– TRU Math –
## The Five Dimensions of Mathematically Powerful Classrooms

<table>
<thead>
<tr>
<th>The Mathematics</th>
<th>Cognitive Demand</th>
<th>Access to Mathematical Content</th>
<th>Agency, Authority, and Identity</th>
<th>Formative Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>The extent to which the mathematics discussed is focused and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained. Students should have opportunities to learn important mathematical content and practices, and to develop productive mathematical habits of mind.</td>
<td>The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students’ mathematical development. There is a happy medium between spoon-feeding mathematics in bite-sized pieces and having the challenges so large that students are lost at sea.</td>
<td>The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematics being addressed by the class. No matter how rich the mathematics being discussed, a classroom in which a small number of students get most of the “air time” is not equitable.</td>
<td>The extent to which students have opportunities to conjecture, explain, make mathematical arguments, and build on one another’s ideas, in ways that contribute to their development of agency (the capacity and willingness to engage mathematically) and authority (recognition for being mathematically solid), resulting in positive identities as doers of mathematics.</td>
<td>The extent to which the teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings. Powerful instruction “meets students where they are” and gives them opportunities to move forward.</td>
</tr>
</tbody>
</table>
What’s New, What’s Different?

In a sense, nothing.
That is,

You should recognize and resonate to everything in TRU.

It captures what we know is important. It doesn’t offer any “magic bullets” or surprises.
So, What’s Different?

TRU is:

- Comprehensive
- Easy to remember
- Easy to work on/with
- It’s a natural frame for PD
There’s one problem with what you’ve seen thus far.
Text is linear, but the ideas aren’t.
Here’s a better image.
TRU:

The Mathematics

Cognitive Demand

Agency Authority Identity

Formative Assessment

Access
Any classroom, from pre-K through graduate school, that does well on these five dimensions, will produce students who are powerful mathematical thinkers.
So much evidence, so little time...

See

http://map.mathshell.org

and

http://ats.berkeley.edu

for evidence, and for the tools I’m about to show you.
Before proceeding, it’s ESSENTIAL to understand:

**TRU** is NOT a tool or set of tools.

TRU is a perspective regarding what counts in instruction, and

**TRU provides a language for talking about instruction in powerful ways.**

If you get this and use it to think about teaching, things can get better.
Part 2:

Tools for supporting powerful classroom instruction:

a. Formative Assessment lessons
b. Planning and Reflection
The Mathematics Assessment Project has created 100 2-3 day lessons that support rich engagement with important mathematical ideas.
Interpreting Distance-Time Graphs

Mathematics Assessment Project
CLASSROOM CHALLENGES
A Formative Assessment Lesson

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley
Beta Version

For more details, visit: http://map.mathshell.org
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detailed at http://creativecommons.org/licenses/by-nc-nd/3.0/ - all other rights reserved
Before the lesson devoted to this topic, we give a diagnostic problem as homework:

Every morning Tom walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.

Describe what may have happened. Is the graph realistic? Explain.
We point to typical student misconceptions and offer suggestions about how to address them...

<table>
<thead>
<tr>
<th>Common Issue</th>
<th>Possible questions and prompts</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Student interprets the graph as a picture</strong></td>
<td>• If a person walked at a steady speed up and down a hill, <em>directly away from home</em>, what would the graph look like?</td>
</tr>
<tr>
<td>E.g. as the graph goes up and down, Tom’s path goes up and down.</td>
<td></td>
</tr>
<tr>
<td><strong>Student interprets graph as speed–time</strong></td>
<td>• How can you tell if Tom is traveling away from or towards home?</td>
</tr>
<tr>
<td>E.g. The student has interpreted a positive slope as speeding up</td>
<td></td>
</tr>
<tr>
<td>and a negative slope as slowing down.</td>
<td></td>
</tr>
<tr>
<td><strong>Student fails to mention distance or time</strong></td>
<td>• Can you provide more information about how far Tom has traveled during different sections of his journey?</td>
</tr>
<tr>
<td>E.g. The student has not worked out the speed of some/all sections of the journey.</td>
<td></td>
</tr>
<tr>
<td><strong>Student fails to calculate and represent speed</strong></td>
<td>• Can you provide information about Tom’s speed for all sections of his journey?</td>
</tr>
<tr>
<td><strong>Student adds little explanation as to why the graph is or is not realistic</strong></td>
<td>• Is Tom’s fastest speed realistic? Is Tom’s slowest speed realistic? Why?/Why not?</td>
</tr>
</tbody>
</table>
The lesson itself begins with a diagnostic task…
Matching a Graph to a Story

A. Tom took his dog for a walk to the park. He set off slowly and then increased his pace. At the park Tom turned around and walked slowly back home.

B. Tom rode his bike east from his home up a steep hill. After a while the slope eased off. At the top he raced down the other side.

C. Tom went for a jog. At the end of his road he bumped into a friend and his pace slowed. When Tom left his friend he walked quickly back home.
Students are given the chance to annotate and explain...

A graph may end up looking like this:

Line not too steep - this means Tom slows down.

Furthest Tom gets from home.

Negative slope means Tom is walking back to his home.

Tom returns home.

Distance from home

Tom starts from home

Time
# Matching stories to graphs - students make posters

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Distance from home vs. time graph]</td>
<td>![Distance from home vs. time graph]</td>
<td>Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.</td>
<td>Opposite Tom’s home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C</th>
<th>D</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Distance from home vs. time graph]</td>
<td>![Distance from home vs. time graph]</td>
<td>Tom skateboarded from his house, gradually building up speed. He slowed down to avoid some rough ground, but then speeded up again.</td>
<td>Tom walked slowly along the road, stopped to look at his watch, realized he was late, and then started running.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>E</th>
<th>F</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Distance from home vs. time graph]</td>
<td>![Distance from home vs. time graph]</td>
<td>Tom left his home for a run, but he was unfit and gradually came to a stop!</td>
<td>Tom walked to the store at the end of his street, bought a newspaper, and then ran all the way back.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>G</th>
<th>H</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Distance from home vs. time graph]</td>
<td>![Distance from home vs. time graph]</td>
<td>Tom went out for a walk with some friends. He suddenly realized he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.</td>
<td>This graph is just plain wrong. How can Tom be in two places at once?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>I</th>
<th>J</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Distance from home vs. time graph]</td>
<td>![Distance from home vs. time graph]</td>
<td>After the party, Tom walked slowly all the way home.</td>
<td>Make up your own story!</td>
</tr>
</tbody>
</table>
Students work on converting graphs to tables:

Whole-class discussion: Interpreting tables (15 minutes)

Bring the class together and give each student a mini-whiteboard, a pen, and an eraser. Display Slide 5 of the projector resource:

**Making Up Data for a Graph**

On your whiteboard, create a table that shows possible times and distances for Tom’s journey.
Tables are added to the card sort...

1. Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.

2. Opposite Tom’s home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.

And the class compares solutions together
5. Tom left his home for a run, but he was unfit and gradually came to a stop!

6. Tom walked to the store at the end of his street, bought a newspaper, then ran all the way back.

7. Tom went out for a walk with some friends when he suddenly realised he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.

8. This graph is just plain wrong. How can Tom be in two places at once?

9. After the party, Tom walked slowly all the way home.

10. Tom jogged to the park, wanted a water, jogged back to the store, then jogged back to the park.
Now, let’s look at this FAL one dimension at a time, to see how the design supports doing well along the 5 dimensions of TRU.
### The Mathematics

*Is the mathematics important, coherent, connected? Are there opportunities for thinking and problem solving?*

The lesson focuses on developing deep understandings of concepts like slope, and its use to describe real world phenomena; it provides opportunities to make connections across different representations (graphs, tables, stories.)
Do the students have opportunities for sense making – for “productive struggle”? 

The card sort and poster activities provide plenty of room for sense making – IF the students are gently scaffolded when they need it. (Remember the list of support questions)
## Access to Mathematical Content

**Who participates, in what ways? Are there opportunities for every student to engage in sense making?**

Whole group conversations, small group work, and student poster presentations provide *opportunities* for teachers to support every student in engaging meaningfully with the mathematics. But . . . this takes hard work, even with the opportunities.
Agentic, Authoritative, and Identificative

Do students have the opportunities to do and talk mathematics? Can they come to see themselves as “math people”?

Whole group conversations, small group work, and student poster presentations provide opportunities for teachers to support every student in building powerful mathematical identities. But . . . this takes hard work, even with the opportunities.
<table>
<thead>
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<tr>
<td>Does classroom discussion reveal what students understand, so that the lesson can be adjusted for purposes of helping students learn?</td>
</tr>
</tbody>
</table>

They’re called **Formative Assessment Lessons** for a reason... 😊
But that’s high school. Surely we can’t do anything remotely like that at the university level.

Actually, there’s a lot we can do, starting with..
... the TRU Math Conversation Guide.

The idea is to exploit the dimensions of TRU Math as arenas for reflecting on one’s teaching – in planning, in reflecting on how things have gone, and in thinking about next steps.
Start with the big ideas:

<table>
<thead>
<tr>
<th>The Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>How do mathematical ideas from this unit/course develop in this lesson/lesson sequence?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cognitive Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>What opportunities do students have to make their own sense of mathematical ideas?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Access to Mathematical Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Who does and does not participate in the mathematical work of the class, and how?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Agency, Authority, and Identity</th>
</tr>
</thead>
<tbody>
<tr>
<td>What opportunities do students have to explain their own and respond to each other's mathematical ideas?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Formative Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>What do we know about each student's current mathematical thinking, and how can we build on it?</td>
</tr>
</tbody>
</table>
and expand them.

**Before a lesson, you can ask:**

- How can I use the five dimensions to enhance my lesson planning?

**After a lesson, you can ask:**

- How well did things go? What can I do better next time?

**Planning next Steps, you can ask:**

- How can I build on what I’ve learned?
There’s a ton of stuff I’m about to zip by, to show you what the conversation guide looks like.

Then I’ll try a quick (and more readable) change of frame.
The Mathematics

Core Question: How do mathematical ideas from this unit/course develop in this lesson/lesson sequence?

Students often experience mathematics as a set of isolated facts, procedures and concepts, to be rehearsed, memorized, and applied. Our goal is to instead give students opportunities to experience mathematics as a coherent and meaningful discipline. This means identifying the important mathematical ideas behind facts and procedures, highlighting connections between skills and concepts, and relating concepts to each other—not just in a single lesson, but also across lessons and units. It also means engaging students with centrally important mathematics in an active way, so that they can make sense of concepts and ideas for themselves and develop robust networks of understanding.

The Mathematics

<table>
<thead>
<tr>
<th>Pre-observation</th>
<th>Reflecting After a Lesson</th>
<th>Planning Next Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>How will important mathematical ideas develop in this lesson and unit?</td>
<td>How did students actually engage with important mathematical ideas in this lesson?</td>
<td>How can we connect the mathematical ideas that surfaced in this lesson to future lessons?</td>
</tr>
</tbody>
</table>

Think about:
- The mathematical goals for the lesson.
- What connections exist among important ideas in this lesson and important ideas in past and future lessons.
- How math procedures in the lesson are justified and connected with important ideas.
- How we see/hear students engage with mathematical ideas during class.
- Which students get to engage deeply with important mathematical ideas.
- How future instruction could create opportunities for more students to engage more deeply with mathematical ideas.
We want students to engage authentically with important mathematical ideas, not simply receive knowledge. This requires students to engage in productive struggle. They need to be supported in these struggles so that they aren’t lost, but at the same time, support should maintain students’ opportunities to grapple with important ideas and difficult problems. Finding a balance is difficult, but our goal is to help students understand the challenges they confront, while leaving them room to make their own sense of those challenges.

### Cognitive Demand

**Core Question:** What opportunities do students have to make their own sense of mathematical ideas

We want students to engage authentically with important mathematical ideas, not simply receive knowledge. This requires students to engage in productive struggle. They need to be supported in these struggles so that they aren’t lost, but at the same time, support should maintain students’ opportunities to grapple with important ideas and difficult problems. Finding a balance is difficult, but our goal is to help students understand the challenges they confront, while leaving them room to make their own sense of those challenges.

### Cognitive Demand

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<tr>
<th>Pre-observation</th>
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<th>Planning Next Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>What opportunities will students have to make their own sense of important mathematical ideas?</td>
<td>What opportunities did students have to make their own sense of important mathematical ideas?</td>
<td>How can we create more opportunities for students to make their own sense of important mathematical ideas?</td>
</tr>
</tbody>
</table>

**Think about:**
- What opportunities exist for students to struggle with mathematical ideas.
- **How students’ struggles may support their engagement with mathematical ideas.**
- **How the teacher responds to students’ struggles and how these responses support students to engage without removing struggles.**
- **What resources (other students, the teacher, notes, texts, technology, manipulatives, various representations, etc.) are available for students to use when they encounter struggles.**
- What resources students actually use and how they might be supported to make better use of resources.
- Which students get to engage deeply with important mathematical ideas.
- **How future instruction could create opportunities for more students to engage more deeply with mathematical ideas.**
- What community norms seem to be evolving around the value of struggle and mistakes.
All students should have access to opportunities to develop their own understandings of rich mathematics, and to build productive mathematical identities. For any number of reasons, it can be extremely difficult to provide this access to everyone, but that doesn’t make it any less important! We want to challenge ourselves to recognize who has access and when. There may be mathematically rich discussions or other mathematically productive activities in the classroom—but who gets to participate in them? Who might benefit from different ways of organizing classroom activity?

<table>
<thead>
<tr>
<th>Access to Mathematical Content</th>
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</thead>
<tbody>
<tr>
<td><strong>Core Question:</strong> Who does and does not participate in the mathematical work of the class, and how</td>
</tr>
</tbody>
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<tr>
<td><strong>Pre-observation</strong></td>
</tr>
<tr>
<td>What opportunities exist for each student to participate in the mathematical work of the class?</td>
</tr>
</tbody>
</table>

**Think about:**
- What range of ways students can and do participate in the mathematical work of the class (talking, writing, leaning in, listening hard; manipulating symbols, making diagrams, interpreting graphs, using manipulatives, connecting different strategies, etc.).
- Which students participate in which ways.
- Which students are most active when, and how we can create opportunities for more students to participate more actively.
- What opportunities various students have to make meaningful mathematical contributions.
- Language demands and the development of students’ academic language.
- How norms (or interactions, or lesson structures, or task structure, or particular representations, etc.) facilitate or inhibit participation for particular students.
- What teacher moves might expand students’ access to meaningful participation (such as modeling ways to participate, providing opportunities for practice, holding students accountable, pointing out students’ successful participation).
- How to support particular students we are concerned about (in relation to learning, issues of safety, participation, etc.).
Agency, Authority, and Identity

Core Question: What opportunities do students have to explain their own and respond to each other’s mathematical ideas?

Many students have negative beliefs about themselves and mathematics, for example, that they are “bad at math,” or that math is just a bunch of facts and formulas that they’re supposed to memorize. Our goal is to support all students—especially those who have not been successful with mathematics in the past—to develop a sense of mathematical agency and authority. We want students to come to see themselves as mathematically capable and competent—not by giving them easy successes, but by engaging them as sense-makers, problem solvers, and creators of mathematical ideas.

### Agency, Authority, and Identity

<table>
<thead>
<tr>
<th>Pre-observation</th>
<th>Reflecting After a Lesson</th>
<th>Planning Next Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>What opportunities exist in the lesson for students to explain their own and respond to each other’s mathematical ideas?</td>
<td>What opportunities did students have to explain their own and respond to each other’s mathematical ideas?</td>
<td>What opportunities can we create in future lessons for more students to explain their own and respond to each other’s mathematical ideas?</td>
</tr>
</tbody>
</table>

**Think about:**
- Who generates the mathematical ideas that get discussed.
- Who evaluates and/or responds to others' ideas.
- How deeply students get to explain their ideas.
- How the teacher responds to student ideas (evaluating, questioning, probing, soliciting responses from other students, etc.).
- How norms around students' and teachers' roles in generating mathematical ideas are developing.
- How norms around what counts as mathematics (justifying, experimenting, practicing, etc.) are developing.
- Which students get to explain their own and respond to others' ideas in a meaningful way.
Formative Assessment

Core Question: What do we know about each student’s current mathematical thinking, and how can we build on it?

We want instruction to be responsive to students’ actual thinking, not just our hopes or assumptions about what they do and don’t understand. It isn’t always easy to know what students are thinking, much less to use this information to shape classroom activities—but we can craft tasks and ask purposeful questions that give us insights into the strategies students are using, the depth of their conceptual understanding, and so on. Our goal is to then use those insights to guide our instruction, not just to fix mistakes but to integrate students’ understandings, partial though they may be, and build on them.

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<tbody>
<tr>
<td><strong>Pre-observation</strong></td>
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<tr>
<td>What do we know about each student’s current mathematical thinking, and how does this lesson build on it?</td>
</tr>
<tr>
<td><strong>Reflecting After a Lesson</strong></td>
</tr>
<tr>
<td>What did we learn in this lesson about each student’s mathematical thinking? How was this thinking built on?</td>
</tr>
<tr>
<td><strong>Planning Next Steps</strong></td>
</tr>
<tr>
<td>Based on what we learned about each student’s mathematical thinking, how can we (1) learn more about it and (2) build on it?</td>
</tr>
</tbody>
</table>

**Think about:**
- What opportunities exist for students to develop their own strategies and approaches.
- What opportunities exist for students to share their mathematical ideas and reasoning, and to connect their ideas to others’.
- **What different ways students get to share their mathematical ideas and reasoning** (writing on paper, speaking, writing on the board, creating diagrams, demonstrating with manipulatives, etc.).
- **Who students get to share their ideas with** (e.g., a partner, the whole class, the teacher).
- How students are likely to make sense of the mathematics in the lesson and what responses might build on that thinking.
- What things we can try (e.g., tasks, lesson structures, questioning prompts such as those in FALs) to surface student thinking, especially the thinking of students whose mathematical ideas we don't know much about yet.
- **What we know and don't know about how each student is making sense of the mathematics we are focusing on.**
- What opportunities exist to build on students’ mathematical thinking, and how teachers and/or other students take up these opportunities.
In schools, we are building “professional learning communities,” where teachers work collaboratively on issues like these. But what if you’re by yourself?

Here’s a simple exercise you can play, in reflecting on your own (or someone else’s) teaching.
Imagine you’re a student in the class.

Think about these questions, from the student perspective:
**Dimension 1:**

<table>
<thead>
<tr>
<th>The Mathematics</th>
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<tr>
<td>• What ideas do I see as the focus of the lesson?</td>
</tr>
<tr>
<td>• What mathematics am I focused on?</td>
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</tbody>
</table>
**Dimension 2:**

<table>
<thead>
<tr>
<th>Cognitive Demand</th>
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</thead>
<tbody>
<tr>
<td>• What opportunities do I have for sense making?</td>
</tr>
<tr>
<td>• How much of the time am I spending:</td>
</tr>
<tr>
<td>- On small tasks</td>
</tr>
<tr>
<td>- Grappling with ideas</td>
</tr>
<tr>
<td>- At sea?</td>
</tr>
</tbody>
</table>
### Dimension 3:

**Access and Equity**

- Are there consistent opportunities for me to engage in meaningful mathematics?
- Can I hide or be ignored?
**Dimension 4:**

**Agency, Authority, and Identity**

- Do I have the opportunity to explain my ideas, and have them recognized?
- How do I feel when the spotlight lands on me?
**Dimension 5:**

<table>
<thead>
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<tr>
<td>• Do classroom discussions reveal my thinking?</td>
</tr>
<tr>
<td>• Does instruction respond to my thinking and help me think more deeply?</td>
</tr>
</tbody>
</table>
Part 3:
And at the University Level??
Part 3: a Q&A with myself...

• Where can I find the time?
see Halmos, “The heart of mathematics”

• What are some easy techniques?
Lecture: 1 minute conversations;
   Clickers, mini-whiteboards
Recitation: Think/Pair/Share
Both: End of period reflections

• What about tasks?
Examine tasks w/r/t the 5 dimensions...
Build tasks with low floors, high ceilings.
Part 3, Continued:

• What about teacher preparation? How much time do you have?
• What about collaborative activities in a department? Check in with Bill Barton & Mike Thomas.
• What else would be useful to hear about? Look into the following:
MAA Invited Paper Session
What Do We Know about University Mathematics Teaching, and How Can It Help Us?

Today, Friday January 8, 1:00 PM - 3:50 PM
Room 607, Washington State Convention Center

Talks by Ann Edwards, Sean Larsen, Greg Oates, Alon Pinto, Chris Rasmussen, John and Annie Selden
Thanks!