Algebraic Explanations: Linking Instruction to Students' Justifications

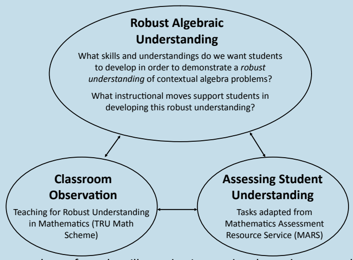
PIs: Dr. Robert Floden, Michigan State University & Dr. Alan Schoenfeld, University of California, Berkeley

Presenters: Jamie Wernet & Jerilynn Lepak, Michigan State University; Sarah Nix & Kimberly Seashore, University of California, Berkeley

NSF DRL-0909851, 0909815

Project Overview

The Algebra Teaching Study (www.ATS.berkeley.edu) is working to develop a classroom observation scheme, TRU Math, in order to capture instructional practices and link them to students' robust understanding of algebra in contextual tasks.



This poster presents a subset of our data illustrating instructional practices around one criterion for robust understanding—the ability to explain and justify algebraic reasoning. We present classroom data from observations in two exemplary 8th grade algebra classrooms and the corresponding assessment of students' written explanations and justifications.

Research questions:

1. How do we capture differences in classroom interactions about explanation and justification?
2. What effect do these interactions have on students' written explanations and justifications?

Methods and Design

Classroom Data

- Recorded 7-8 lessons from ten 8th-grade algebra classes in Michigan and California using field notes and video.
- Coded three representative lessons from two classrooms for instruction and student practice around explanations and justifications using three rubrics (below) from the TRU Math scheme.

Student Data

- Students completed pre- and post-assessments consisting of tasks that provide opportunities for students to explain and justify reasoning.
- Rubrics were created and used to score student work that captured evidence of students' robust understanding – credit was given for Level 2 or 3 explanations.

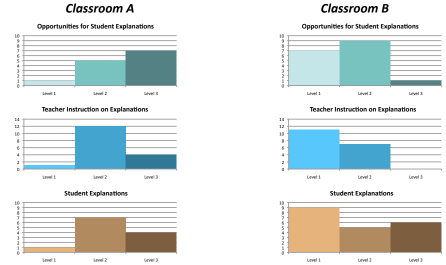
TRU Math Explanation Rubrics

	Level 1	Level 2	Level 3
Opportunities for Student Explanations	The task or teacher opens up space for explanation, but does not specifically require an explanation	The task or teacher explicitly request an explanation, but the nature of the explanation is not specific; does not necessarily require an algebraic justification	The task or teacher explicitly request an explanation focusing on conceptual explanation or justification of algebraic reasoning
Teacher Instruction on Explanations	Teacher explicitly models an explanation by describing "why?" rather than "how?" the math works	Teacher explicitly provides general guidelines on what is needed for good explanations	Teacher provides feedback and/or opportunities for students to revise specific explanations
Student Explanations in Whole Class Discussion	Student generates a short explanation that refers only to procedures	Students generate an explanation using procedures and/or contextual reasoning	Students generate a cogent explanation with sentences drawing on procedures and/or context, including generalized reasoning or inferences from known mathematics

TRU Math: Six Dimensions for Analysis

- **Mathematical Focus:** Is the mathematics discussed focused, coherent, and connected?
- **Cognitive Demand:** Do the student have the opportunity to engage in "productive struggle"?
- **Access:** Who has opportunities to engage in the classroom discourse? Is attention paid to each student's contributions?
- **Agency, Accountability, and Authority:** How are student ideas taken up and attributed in the classroom?
- **Uses of Assessment:** How is student thinking elicited and built upon?
- **Development of *Robust Understanding of Algebra* (see right):** Are students learning specific aspects of interpreting, representing, and explaining when solving contextual algebra problems?

Classroom Profiles of Explanations



Classroom Examples

	Classroom A	Classroom B
Opportunities for Student Explanations	"I'm asking you guys to rewrite the manual to make it better, by giving an explanation in words that says how and why, a labeled picture that shows how and why, and an equation that says how and why." **(Level 3)**	"Does it make sense to you that if it takes Caroline 6 hours and it takes Georgia 4 hours to paint the fence, that it will take them 5 hours together. Does that make sense to you? I heard you say 'no', I want to hear *why*." **(Level 2)**
Teacher Instruction on Explanations	"You open the manual to see how many tiles are gonna be needed and this is the explanation you're given. And it's like a so-so explanation - there are some things, and other things are missing. But based on what you have learned in a math class you need all of these things [points to *words, pictures, equations* written on the board] and not only do you need them, but you need to explain *why*." **(Level 3)**	"The first big idea is that when people work together, the job will take *less* time than for either of the people to do the work alone. That's why if they work together, it has to be less than 4 hours" [teacher points to the board at the time for the faster person]. **(Level 1)**
Student Explanations in Whole Class Discussion	Teacher A: Jacob says they're equal…Jasmine says they're not. They cant both be correct; who's correct? Student: "Jasmine is correct, because if the expressions are equivalent, they have to be equal all the time." **(Level 3)**	Teacher B: [calling on a student] "We haven't heard from your team yet." Student: "I think that the average is 5, that part's right, but if they are doing it together, you put it over two. Because usually if it is just one person then you put it over one. Because if they are doing it together, one does one half and the other does the other half, you go in the middle." **(Level 3)**

Robust Student Understanding

Five criteria are used to assess growth in students' abilities to solve contextual problems in algebra and capture the quality of classroom interactions:

1) Interpreting Context: Students are able to interpret a problem statement to make sense of the problem situation.

2) Identifying Relevant Quantities and Relationships:

a) Students are able to identify which quantities are relevant to the problem situations.

b) Students are able to articulate the mathematical relationships between quantities.

3) Representing Quantitative Relationships:

a) Students are able to generate appropriate algebraic representations.

b) Students are able to interpret and make connections between representations.

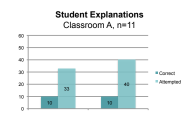
4) Executing Procedures and Checking Solutions:

a) Students are able to execute algebraic procedures and arithmetic calculations.

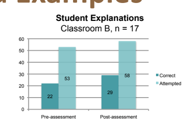
b) Students check the plausibility of their results by attending to the problem context and considering their solution methods.

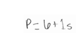
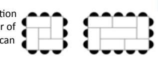
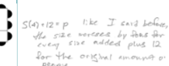
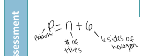
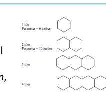
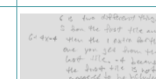
5) Explaining and Justifying Reasoning: Students are able to clearly and thoroughly explain and justify their reasoning. (This poster)

Assessment Results and Examples



Students in both classes made modest gains in their explanation scores: Class A from 11.5% to 13%, and Class B from 17.1% to 23.6% of explanations correct. The number of attempted explanations also increased in both classes.



	Student X Classroom A	Assessment Prompt	Student Y Classroom B
Pre-assessment		Write an equation for the number of people *p* who can sit at a Size *S* arrangement.  **Explain how the parts of your equation relate to the table arrangement.**	
Post-assessment		Write an equation for the perimeter *p* of a row of hexagonal tiles that works for any number of tiles, *n*, in the row.  **Explain how the parts of your equation relate to the hexagon patterns.**	 "6 is two different things. 5 from the first tile and then 1 extra for the one you get from the last tile. - 4 because the first tile is not supposed to be included.

Preliminary Findings and Future Work

- To illustrate the work of ATS, we showed how we can capture elements of classroom interactions related to explanations and justifications and build profiles of two algebra classrooms with teachers identified as highly effective. The profiles indicate that, while there are different emphases, students in both classes had opportunities for instruction and practice on providing explanations.
- TRU Math is designed to capture variation in classroom interactions and instruction. Our goal is for the scheme to be sensitive enough to capture variation while reflecting the hypotheses we have regarding practices that will lead to robust algebraic understanding.
- We plan to use the scheme to create profiles of large numbers of algebra 1 classrooms to provide empirical support for classroom interactions that support the development of robust algebraic understanding.