



TRANSITIONING FROM EXECUTING PROCEDURES TO ROBUST UNDERSTANDING OF ALGEBRA

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Algebra Teaching Study

Michigan State University and UC-Berkeley California.

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Algebra Teaching Study Goals

- Overall aim is to provide research tools
 - Student assessment
 - Classroom observation scheme
- Focus: Robust understanding for contextual problems in algebra
- Better understanding of connections between classroom instruction and students' robust understanding

Development of Tools

Student Assessments

- Selected and revised tasks from the Mathematics Assessment Resource Service (MARS, <http://www.noycefdn.org/resources.php>)
- Constructed assessments
 - Five multiple choice questions
 - Three open-ended contextual tasks
- Developed scoring rubrics to capture detailed student competencies

Observation Scheme

- *Teaching for Robust Understanding of Mathematics* (TRU MATH)
- Developed iteratively over four years (still in progress) by watching classroom video, and identifying promising classroom practices
- Two parts
 - Five Dimensions
 - Algebra-specific practices

Robustness Criteria for Algebraic Understanding (RCs)

RC1: Reading and interpreting text; understanding context

RC2: Identifying salient quantities and relationships between them

RC3: Algebraic representations of relationships between quantities

RC4: Executing calculations and algebraic procedures

RC5: Explaining and justifying reasoning

2a. Identify relevant quantities

2b. Articulate relationships between quantities

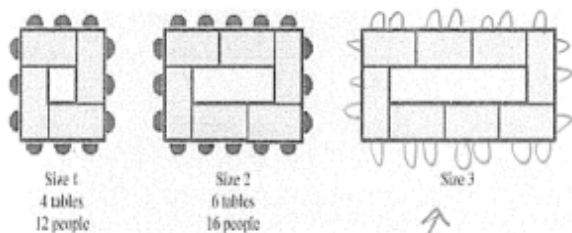
3a. Generate representations

3b. Interpret and connect representations

4a. Execute calculations and procedures with precision

4b. Check plausibility of results

Capturing RC 3 in Sample Student Work



(1) How many people can sit at a Size 3 arrangement?

20 people

(2) How many people can sit around a Size 13 arrangement? Explain how you know your answer is correct.

Size 13
28 tables
60 people

(size)	Size 1	Size 2	Size 3	Size 4	5	6	7	8	9	10	11	12	13
(table)	4	6	8	10	12	14	16	18	20	22	24	26	28
(people)	12	16	20	24	28	32	36	40	44	48	52	56	60

I know this is right because the chart helped me.

(3) Write an equation for the number of people p who can sit at a Size S arrangement. Explain how the parts of your equation relate to the table arrangements.

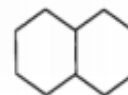
$$p = S + t$$

↑ ↑ ↑
 this has to do with people this has to do with tables this has to do with size.

1 tile
Perimeter = 6 inches



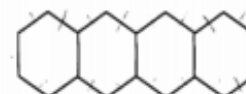
2 tiles
Perimeter = 10 inches



3 tiles



4 tiles

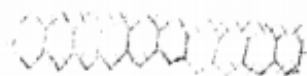


(1) Find the perimeter of her arrangement of 4 tiles.

18 inches

(2) What is the perimeter of a row of 10 tiles? How do you know this is the correct perimeter for 10 tiles?

42 inches. I know this because the perimeter = the # of hexagons $\times 6 - (\# \text{ of hexagons} - 1 \times 2)$



$$10 \times 6 - 9 \times 2$$

$$60 - 18 = 42$$

(3) Write an equation for the perimeter p of a row of hexagonal tiles that works for any number of tiles, n , in the row. Explain how the parts of your equation relate to the hexagon patterns on the first page.

$$p = n \times 6 - (n - 1) \times 2$$

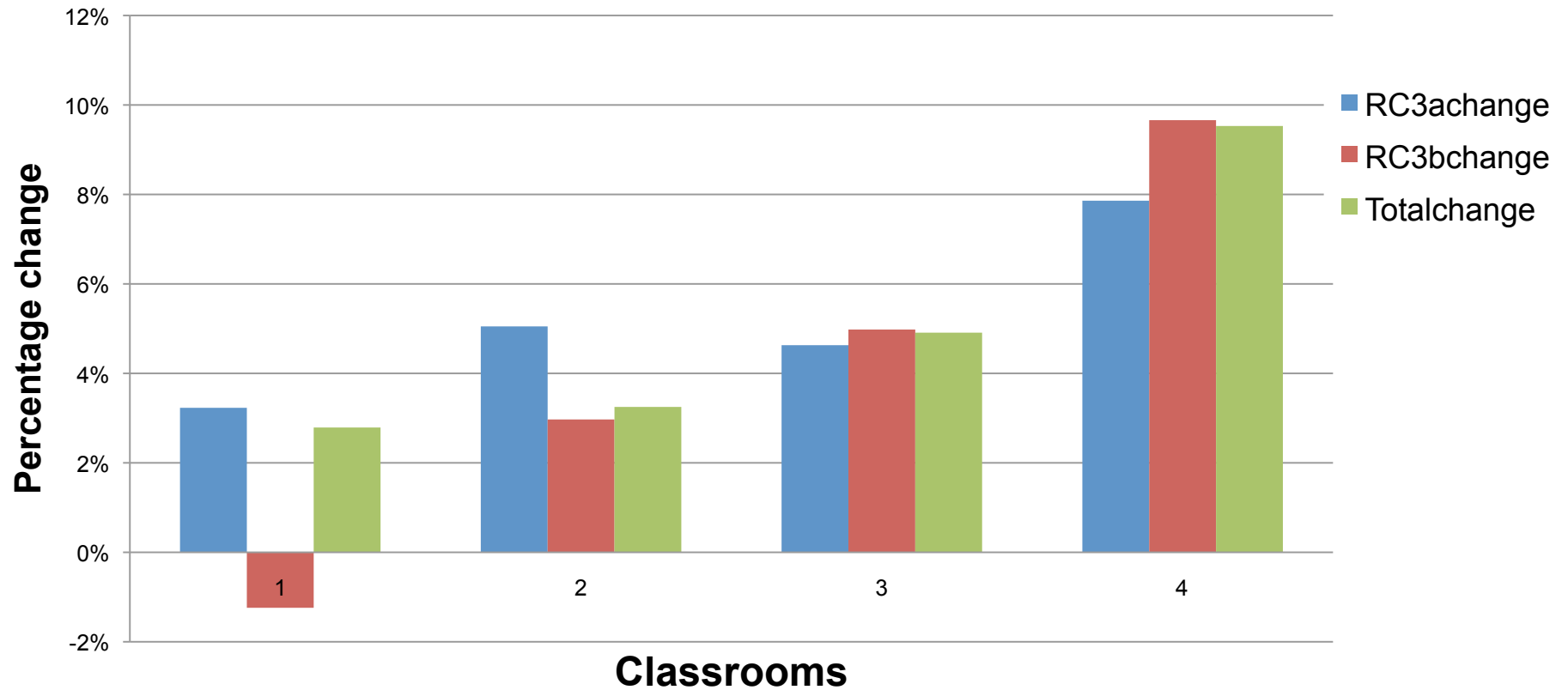
↑ ↑
 6 sides to the hexagon taking away the sides that are connected together.

RC 3 Rubrics from Scheme

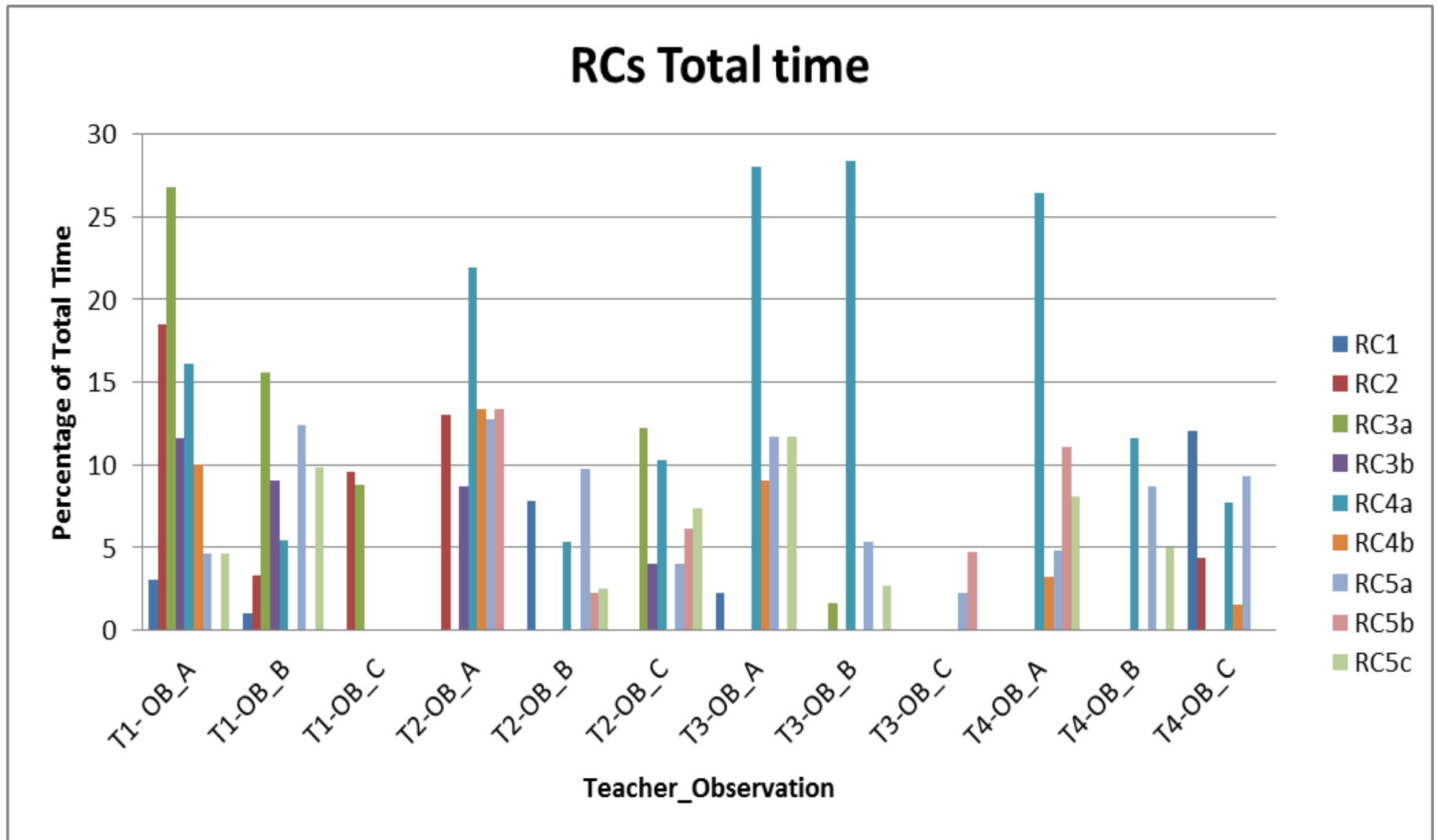
		Level 1	Level 2	Level 3
RC 3a	Generating representations of relationships between quantities	Generated by way of practice	Purposefully generated with explicit attention to the relationship between variables	Level 2 <i>and</i> attention to why the representation is a good choice for the given situation
RC 3b	Interpreting and making connections between representations	Representations are interpreted locally or in part . No connections between multiple representations.	Global features of representations are explicated to highlight the covariation between quantities	Level 2 <i>and</i> connections among multiple representations are explored

Tracking Changes in Student Scores Across Four Classrooms

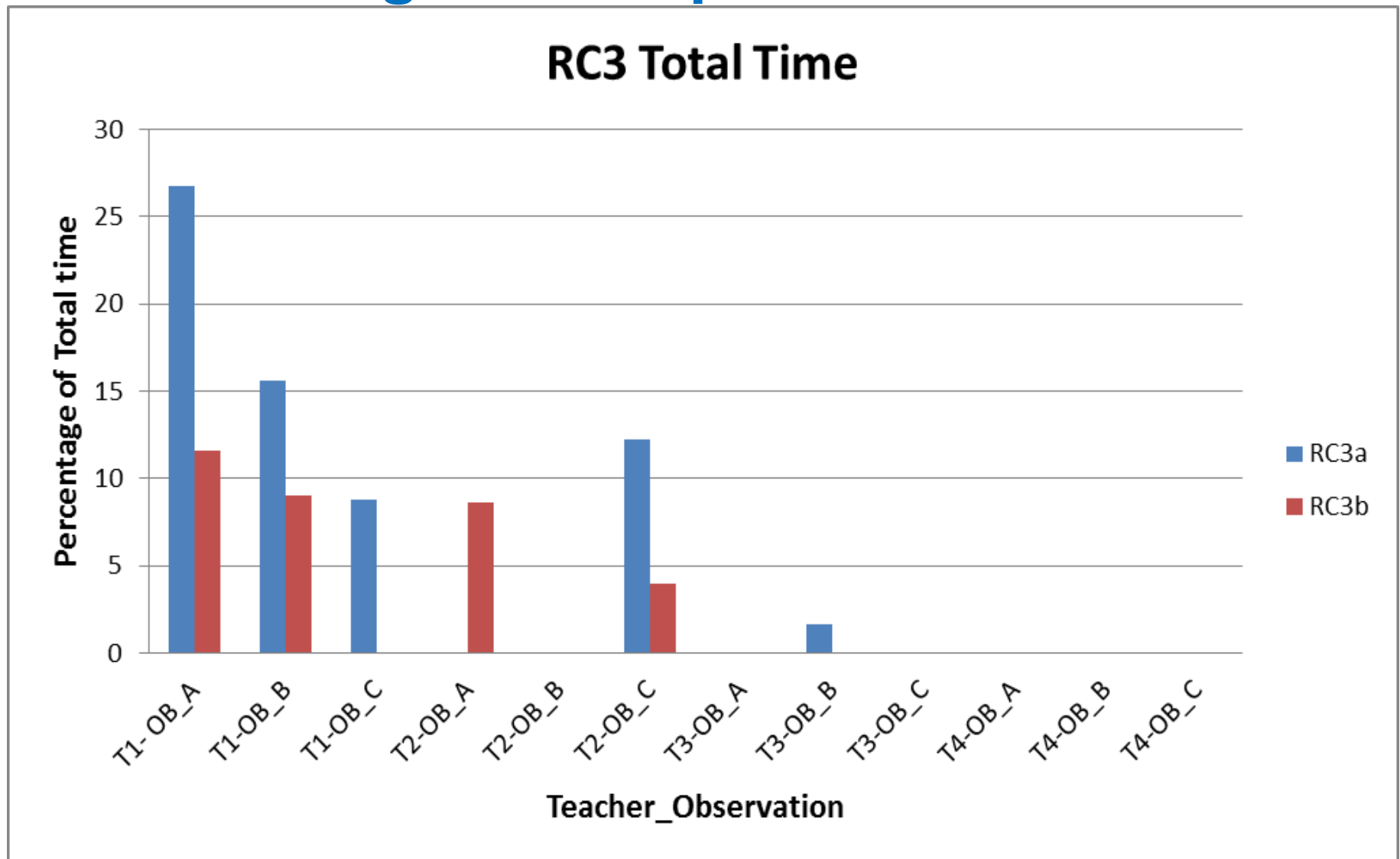
Mean Score Change



Tracking Classroom Practices



Tracking Generation and Interpretation of Algebraic Representations



Discussion

The tools in development can be used to:

- Track how often a class addresses RC-related proficiencies in a lesson and across lessons, and the nature of that attention.
- Measure student understanding of a detailed set of skills and competencies.
- Begin to highlight differences in teachers' use of instructional practices and students' robust understanding of algebra across classrooms.

The development of tools that can link student learning to classroom practices is an important step in identifying effective practices that lead to robust understanding of mathematics.

Thank you!

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Please visit our two project posters (one on the scoring rubrics and one on the scheme) later today!